Hydrostatic Pressure and Magnetic Field Effects on the Exciton States in Vertically Coupled GaAs-(Ga, Al) As Quantum Dots

M. E. Mora-Ramos\textsuperscript{1}, A. H. Rodríguez\textsuperscript{2}, S. Y. López\textsuperscript{3}, and C. A. Duque\textsuperscript{4,\textsuperscript{5}}

\textsuperscript{1}Universidad Autónoma del Estado de Morelos, Ave. Universidad 1001, 62209 Cuernavaca, México
\textsuperscript{2}Universidad Autónoma de la Ciudad de México, Plantel Iztapalapa, México DF, México
\textsuperscript{3}Facultad de Educación, Universidad de Antioquia, AA 1226, Medellín, Colombia
\textsuperscript{4}Instituto de Física, Universidad de Antioquia, AA 1226, Medellín, Colombia
\textsuperscript{5}Instituto de Física, Unicamp, CP 6165, Campinas-SP, 13083-970, Brazil

Abstract — The variational procedure, in the effective-mass and parabolic-band approximations, is used in order to investigate the combined effects of hydrostatic pressure and in-plane-direction-applied magnetic field on the exciton states in vertically coupled GaAs-(Ga, Al) As quantum dots. Calculations are performed for two cylindrical-shape quantum dots. The exciton envelope wave function is obtained through a variational procedure using a hydrogenic 1s-like wave function and an expansion in a complete set of trigonometric functions for the electron and hole wave functions. The anticrossing effects on the dispersion with applied magnetic field and hydrostatic pressure of the photoluminescence peaks associated with direct and indirect excitons have been considered.

1. INTRODUCTION

A symmetric/asymmetric coupled double quantum well (DQW) is made of two identical/different quantum wells (QW) that are separated by a thin barrier. For the symmetric case, in flat-band conditions, i.e., without applied electric field in the growth direction of the heterostructure, the eigenfunctions of the DQW have well-defined symmetries. These are broken in the asymmetric case. In this case, only transitions between electron and hole states with the same symmetry are optically allowed. Whenever the maximum probability of the electron and hole wave functions are distributed in the same well, the transitions are known as spatially direct transitions. The intensity of these optical transitions is essentially given by the overlap integral of the electron and hole single-particle envelope wave functions and temperature-dependent populations of electrons and holes in the subbands. Also, it is certainly necessary to take into account the electron-hole \( e - h \) Coulomb interaction for an appropriate description of the optical transitions in semiconducting heterostructures. Of course, effects of the \( e - h \) Coulomb interaction are essential whenever the fine structure of the optical spectra shows features which are within the range of the exciton binding energy. On the other hand, the application of hydrostatic pressure results in changes of the dielectric constant and of band structure parameters such as the energy gap and the conduction band mass. This may result in modifications of the interband optical transitions in GaAs-based QW’s (see Ref. [1], and references therein).

By applying an in-plane magnetic field in coupled QWs, it is possible to induce strong changes in the excitonic-related photoluminescence (PL) spectra due to field-induced displacement of the interwell exciton dispersion in momentum space, which leads to a transition from the momentum-space direct exciton ground state to the momentum-space indirect exciton ground state [2–4]. The indirect exciton lifetime in coupled DQW heterostructures under applied magnetic fields has been studied by Butov et al [2–4] and they attribute the observed results to an increase in the magnetoexciton mass. Also, Butov et al [2–4] have studied long-lifetime indirect excitons in coupled QWs and, at low temperatures and high exciton densities, strong deviations of the indirect exciton PL kinetics from monoexponential PL rise/decay were observed. Parlangeli et al [5] have studied the indirect exciton dispersion in \( k \) space by considering the simultaneous effect of in-growth direction applied electric field and in-plane magnetic field in DQW heterostructures and found that the PL spectra increase with the magnetic field following a quadratic behavior. Additionally, they present measurements of the PL peak positions of both direct and indirect excitons in biased GaAs/Ga\(_{1-x}\)Al\(_x\)As coupled DQWs under in-plane applied magnetic fields.

In the present work we are concerned with a theoretical study of the effects of applied hydrostatic pressure and in-plane magnetic fields on the exciton direct and indirect states in GaAs/Ga\(_{1-x}\)Al\(_x\)As
vertically coupled double quantum dots (CDQD). The potential function profile of this kind of systems is mathematically treated in a way that is very similar to that of the double QW. The theoretical framework is outlined in Section 2. Results and discussion are presented in Section 3 and finally in Section 4 we outline our conclusions.

2. THEORETICAL FRAMEWORK

The theoretical approach assumes the envelope-function and parabolic-band approximations [6]. We choose the reference system at the barrier center, with the \( z \)-axis along the growth direction of the structure, the in-plane magnetic field in the \( x \)-direction, \( \mathbf{B} = B\hat{x} \), and use the Landau gauge \( \mathbf{A}(\mathbf{r}) = -B\hat{z}\hat{y} \). The Hamiltonian for the exciton then takes the following form [7-9]

\[
\hat{H} = \frac{1}{2m_e^*} \left( \mathbf{p}_e + \frac{e}{c} \mathbf{A}_e \right)^2 + \frac{1}{2m_h^*} \left( \mathbf{p}_h - \frac{e}{c} \mathbf{A}_h \right)^2 + V_e(z_e) + V_h(z_h) + V_e(\rho_e) + V_h(\rho_h) - \frac{e^2}{\epsilon |\mathbf{r}_e - \mathbf{r}_h|},
\]

where \( \mathbf{A}_e = \mathbf{A}(\mathbf{r}_e) \), \( \mathbf{A}_h = \mathbf{A}(\mathbf{r}_h) \), and \( \mathbf{p}_i, \mathbf{r}_i, m_i^* \) and \( V_i \), with \( i = e, h \), are the momentum operators, electron and hole coordinates, effective masses and corresponding CDQD confining potentials, respectively, \( e \) is the absolute value of the electron charge and \( \epsilon \) is the GaAs dielectric constant. For simplicity, the dielectric constant and the effective masses are considered the same as in GaAs throughout the GaAs-Ga\(_{1-x}\)Al\(_x\)As CDQW. The dependencies on hydrostatic pressure of conduction band mass, barrier heights, and the dielectric constant are introduced according to Ref. [1].

In our model we consider two cylindrical quantum dots vertically coupled by a finite potential barrier. In the external walls of each quantum dot we have considered an infinite potential barrier. In the case of the radial confinement, the potentials that we consider are also infinite. It is for that reason that in the Eq. 1, both for the electron and the hole, the potential that confines them can be written as a sum of two potentials the first one in the \( z \)-direction and the second one in the \( \rho \)-direction. In the case in which we consider finite barriers in all direction, this separation of the potential in a sum of potentials only will be valid for large quantum dots where the wave functions of each particle have a little contribution in the regions of the barriers.

In order to obtain the exciton eigenfunctions for the GaAs-Ga\(_{1-x}\)Al\(_x\)As CDQD, we adopt the variational scheme which consists in minimizing the functional

\[
E(\Phi) = \langle \Phi | \hat{H} | \Phi \rangle
\]

by using the variational wave functions as

\[
\Phi(\rho, z_e, z_h) = N f(z_e)F(z_h)g(\rho_e)g(\rho_h) e^{-\lambda r},
\]

where \( r = \sqrt{\rho^2 + (z_e - z_h)^2} \), \( \lambda \) is a variational parameter, \( f(z_e) \) and \( F(z_h) \) are, in general, linear combinations of the \( z \)-dependent part of the electron \( f_i(z_e) \) and hole \( F_j(z_h) \) eigenfunctions of the total Hamiltonian neglecting the Coulomb interaction [10], \( g(\rho_e) \) and \( g(\rho_h) \) are the corresponding in-plane wave functions. The coefficients \( a_i^{(e)} \) and \( b_j^{(h)} \) of above mentioned linear combinations are also variational parameters satisfying the usual normalization conditions. Finally, in order to obtain the non-correlated \( f_i(z_e) \) electron and \( F_j(z_h) \) hole eigenfunctions, it is convenient to use the method by Xia and Fan [11] of expansion in terms of sine functions, used in the study of electron states in semiconductor superlattices in the presence of in-plane magnetic fields. In the variational approach described above, the effect of the Coulomb interaction is to mix the GaAs-Ga\(_{1-x}\)Al\(_x\)As CDQD electron and hole-wave functions \( f_i(z_e) \) and \( F_j(z_h) \), respectively. Here we are interested in excitons associated to the GaAs-Ga\(_{1-x}\)Al\(_x\)As CDQD ground state, and limit ourselves to the cases for which only the mixing between the CDQD electron ground state \( f_0(z_e) \) and electron first-excited state \( f_1(z_e) \) is important, whereas mixing effects for the CDQD hole states are disregarded. The corresponding variational exciton wave functions then take the form

\[
\Phi_+(\rho, z_e, z_h) = \left[ \alpha f_0(z_e) + \sqrt{1 - \alpha^2} f_1(z_e) \right] F_0(z_h)g(\rho_e)g(\rho_h) e^{-\lambda_+ r}
\]

and

\[
\Phi_-(\rho, z_e, z_h) = \left[ -\sqrt{1 - \alpha^2} f_0(z_e) + \alpha f_1(z_e) \right] F_0(z_h)g(\rho_e)g(\rho_h) e^{-\lambda_- r},
\]
where \( F_0(z_h) \) is the CDQD hole ground state, \( \alpha, \lambda_+ \) and \( \lambda_- \) are variational parameters, and we follow the procedure by Fox et al [10] in the process of minimizing \( E(\Phi) \) [cf. Eq. (2)], using the wave functions (4) and (5).

3. RESULTS AND DISCUSSION

In Figure 1 we present our results for the in-plane applied magnetic field dependence of the calculated PL peak transitions for two vertically coupled cylindrical quantum dots. Results are for two different values of the hydrostatic pressure. Clearly, there is a quadratic behavior as a function of the applied magnetic field. The vertices of the parabolas are shifted in energy as an effect of the vertical and radial confinement. As the magnetic field grows the influence of the potential barrier, which separates the two quantum dots, decreases, and the energy curves go to a linear behavior determined by the first Landau level. This level is displaced in a value of the energy determined by the radial confinement. We note that the energy curves grow with the increasing width of the potential barriers. This is due to the fact that barrier width increasing implies that the two quantum dots are more and more isolated and the carriers become essentially confined in the region of a single quantum dot. Accordingly, the energy of each particle (electron or hole) becomes higher and, as a result, the energy of the PL-peak increases.

![Figure 1](image_url)

**Figure 1**: In-plane applied magnetic field dependence of the calculated PL peak for the transitions between the first coulomb-perturbed electron states and the first heavy-hole level for a system of two couple cylindrical quantum dots with radius \( R \), heights \( L_1 \) and \( L_2 \), separated by a barrier of width \( L_b \). Hydrostatic pressure. (a) \( P = 0 \), and (b) 50 kbar.

In the Figure 1(b) we have considered the dimensions of the structures, but with the additional effect of a pressure of 50 kbar. In essence, the effect of the latter is seen as a blue-shift in the PL-peaks. This large blue-shift is mainly due to the dependence with the pressure of the band-gap of the quantum dot material (GaAs). The dependencies with the pressure of the dimensions of the structure modify in an non-significant value the PL-peak energies (changes are smaller than 1 %). The changes in the effective masses and the static dielectric constant with the pressure modify the binding energy leading to its increasing with the pressure, which manifests as a red-shift in the PL-peak. Indeed, this shift is overlapped by the dependence with the pressure of the energy gap of the GaAs. Finally, we observe that the applied pressure makes the effects of the magnetic field to become greater. This fact is associated with the decrease of the height of the central barrier and with the increasing in the effective mass of the electrons.

4. CONCLUSIONS

The variational procedure, in the effective-mass and parabolic-band approximations, have been used in order to investigate the combined effects of hydrostatic pressure and in-plane-direction-applied magnetic field on the exciton states in vertically coupled GaAs-(Ga,Al) As quantum dots. Calculations are performed for two cylindrical-shape quantum dots. We have observed a quadratic dependence with the pressure of the PL-peak energy. However, for large magnetic field values, a
linear behavior can be predicted, in accordance with the variation of the Landau levels due to the quantum confinement. Additionally, we observe that the magnetic field effects are magnified when a hydrostatic pressure is considered on the structure.

ACKNOWLEDGMENT

M. E. M. R. acknowledges support from Mexican CONACYT through grant No. 52304. C. A. D. and S. Y. L. would like to thank the Colombian CODI-Univ. de Antioquia for partial financial support. This work was also partially financed by the Excellence Center for Novel Materials and COLCIENCIAS under contract No 043-2005.

REFERENCES